STRAIGHT LINES SYNOPSIS

Basic two dimensional geometry concepts:

2.Triangle: (Centroid)

a). Centroid of the triangle ABC $A(x_1, y_1)\ B(x_2, y_2)\ C(x_3, y_3)$ with vertices, is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

b) Centroid divides the line segment joining a vertex and mid point of the side opposite to it, in the ratio 1:2 (i.e. AG:GD::2:1)

c) In a triangle ABC, $AB^2 + BC^2 + AC^2 = 3\left(GA^2 + GB^2 + GC^2\right)$, where G is centroid of the triangle ABC.

d) In a triangle ABC, $3\left(AB^2 + BC^2 + AC^2\right) = 4\left(AD^2 + BE^2 + CF^2\right)$, where D,E and F are the mid points of the side BC, CA and AB respectively.

e) If P is any point in the plane of triangle ABC and G is the centroid then $PA^2 + PB^2 + PC^2 = GA^2 + GB^2 + GC^2 + 3PG^2$

f) If G is the centroid of the triangle ABC, then

area of the triangle ABC = 3(area of the triangle AGB) = 3 (area of the triangle BGC)= 3(area of the triangle ACG)

g) D,E and F are the mid points of the sides of the triangle BC, CA and AB respectively then

i) Centoid of the triangle ABC = centroid of the triangle DEF

ii) area of the triangle BDF = area of the triangle DEC = area of the triangle DEF = area of the triangle AEF= ¼ (area of the triangle ABC)

h) If the sides BC,CA and AB of a triangle ABC are divided by the points D,E and F in the same ratio, then centroid of the triangle ABC and DEF are coincide.

i) In any triangle ABC, $AB^2 + AC^2 = 2(AD^2 + BD^2)$, where D is the mid point of the side BC.

k) Length of the median drawn through the vertex A is $\sqrt{\frac{2b^2 + 2c^2 - a^2}{4}}$

3.Incentre/cicumcentre/orthcentre

a) Incentre of the triangle ABC $A(x_1, y_1)\ B(x_2, y_2)\ C(x_3, y_3)$ with vertices and AB = c, BC = a, CA = b, is $\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + ay_2 + ay_3}{a+b+c}\right)$,

b) Incentre of the triangle ABC $A(x_1, y_1)\ B(x_2, y_2)\ C(x_3, y_3)$ with vertices and AB = c,BC = a,CA= b, is I and AI:ID=b+c:a,

c) In a triangle ABC, if orthocenter is O, then the four points O,A,B,C are such that each point is orthocenter of the triangle formed by the remaining three points.

d) In any triangle, circumcentre, centroid & orthocenter will be collinear

e) In an isosceles triangle centroid, circumcentre, Incentre and orthocentre are collinear points.

f) In any equilateral triangle, centroid, circumcentre, Incentre and orthocenter all coincide.

g) The line segment, joining circumcentre and orthocenter is divided by centroid in the ratio 1 : 2 internally CG:GO::1:2

5.Translation and Rotation:

a) If P (x, y) are coordinates of a point, referred to the co ordinate axes x and y with origin (0,0) and (X, Y) coordinates of the same point P referred to the new axes X and Y

If origin is shifted to (h, k) then $x = X + h$ and $y = Y + k$

6.Some standard locus:

a). A, B are two given points, "p" is any point

Minimum of $PA+PB$ is AB. And the point "P" for which $PA+PB$ is minimum, obtained when P, A, B are collinear and P lies between A and B.

b) A, B are two given points, "p" is any point, such that $PA+PB$ = k,

If k = AB, then locus of p is a line segment AB

If k > AB, then locus of p Ellipse, with A, B as foci & eccentricity as $\frac{AB}{k}$

If k < AB, then locus of p does not exist.

c). A, B are two given points, "p" is any point . Maximum of $|PA-PB|$ is AB. And the point "P" for which $|PA-PB|$ is maximum, is obtained when P, A, B are collinear, P lies outside the line segment AB.

d) A, B are two given points, "p" is any point, such that $|PA-PB|$ = k,

If k = AB, then locus of p is a pair of rays

If k > AB, then locus of p does not exist

If k < AB, then locus of p is Hyperbola, with A, B as foci, eccentricity as $\frac{AB}{k}$

e). If A,B are two fixed points and P is a moving point such that $PA = n\,PB\,(n \neq o).$ then

ei). if $|n| = 1,$ then locus of P is a line, which is perpendicular bisector of line segment AB

eii). if $|n| \neq 1$ the locus of P is a circle, described on CD as diameter where C, D are points dividing AB in the ratio n : 1 internally, externally respectively. (the circle never pass through A & B).

eiii). if $0 < n < 1$ then locus of P is a circle and A lies inside and B lies outside the circle

eiv). if $n > 1$ then locus of P is a circle and A lies outside and B lies inside the circle

STRAIGHT LINES:

b) $y = mx + c \ (m, slope, \ \ c, y-\text{int}ercept)$

d). $y - y_1 = m(x - x_1)$ is equation of the line, passing through the point (x_1, y_1) and having slope m e). $\frac{y-y_1}{y_1-y_2}=\frac{x-x_1}{x_1-x_2}$ is the equation of the line passing through two given points $(x_1,y_1),\ (x_2,y_2)$

f). Equation of the line which makes a and b as intercepts on x-axis and y-axisis given by $\frac{x}{a}+\frac{y}{b}=1.$

g). Equation of the line in normal form is given by $x\cos\alpha+y\sin\alpha=p.$ where "p", distance from origin to the line, $\alpha -$ angle between the x- axis, & the line drawn from origin and perpendicular to the given line. $\alpha\in[0,\ 2\pi)$

h). Equation of the line in parametric form is given by $\frac{x-x_1}{\cos\theta}=\frac{y-y_1}{\sin\theta}=r.$

Any point on the line, which is at a distance of "r" units away from P $(x_1,\ y_1)$ is Q$(x_1+r\cos\theta,\ y_1+r\sin\theta).$ r is $+ve$ if the point Q is relatively above the point P (x_1,y_1) and $r=-ve$ if the point Q is relatively below the point P (x_1,y_1)

8.Position of two points, with respect to a given line $L_1:\ ax+by+c=0.$

a). Suppose $P(x_1,y_1),\ Q(x_2,\ y_2)$ lie on the same side of the given line $\Rightarrow L_1(P).L_1(Q)>0$

b). Suppose $P(x_1,y_1),\ Q(x_2,\ y_2)$ lie on the opposite side of the given line $\Rightarrow L_1(P).L_1(Q)<0$

9.AREA

a). Area of triangle with vertices $(x_1,y_1)(x_2,\ y_2)(x_3,y_3)$ is $\frac{1}{2}\begin{vmatrix} x_1-x_3 & x_2-x_3 \\ y_1-y_3 & y_2-y_3 \end{vmatrix}.$

c). Area of a polygon of "n" sides whose vertices, are given by

$(x_i,y_i),i=1,2,......n,\ is\ \frac{1}{2}\left\{\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}+\begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix}+......+\begin{vmatrix} x_n & y_n \\ x_1 & y_1 \end{vmatrix}\right\}$

d). Area of a triangle, formed by the lines $a_ix+b_iy+c_i=0\ \ i=1,2,3.$

is $\frac{1}{2|c_1c_2c_3|}\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^2$ where $\left.\begin{matrix} C_1=a_1b_2-a_2b_1 \\ C_2=a_2b_3-a_3b_2 \\ C_3=a_3b_1-a_1b_3. \end{matrix}\right\}$ co factors of $c_1,\ c_2,\ c_3$ and $y=m_ix+c_i\ \ i=1,2,3.$

e) Area of triangle formed by the lines $y=m_ix+c_i,\ \ i=1,2,3.\ is\ \frac{1}{2}\left|\frac{(c_1-c_2)^2}{m_1-m_2}+\frac{(c_2-c_3)}{m_2-m_3}+\frac{(c_3-c_1)^2}{m_3-m_1}\right|.$

g). Area of a parallelogram, formed by $a_1x+b_1y+c_1=0$, $a_1x+b_1y+c_2=0$, $a_2+b_2y+d_1=0$ and $a_2x+b_2y+d_2=0$ is $=\left|\frac{P_1P_2}{\sin\theta}\right|$. where P_1, P_2 are the distance between the pair of parallel lines and θ is angle between the lines, $\left|\frac{P_1P_2}{\sin\theta}\right| = \frac{|c_1-c_2||d_1-d_2|}{|a_1b_2-b_1a_2|}$ $\left[?\tan\theta = \frac{a_2b_1-a_1b_2}{a_1a_2+b_1b_2},\ \sin\theta = \frac{a_2b_1-a_1b_2}{\sqrt{a_1^2+b_1^2}\sqrt{a_2^2+b_2^2}.}\right]$

& $P_1 = \frac{c_1-c_2}{\sqrt{a_1^2+b_1^2}}.\ P_2 = \frac{d_1-d_2}{\sqrt{a_2^2+b_2^2}}.$

Examples: 14). Area of a rhombus, formed by the lines $ax \pm by \pm c = 0\ is \left|\frac{2c^2}{ab}\right|$

11.Concurrency of three lines:

a) Three or more lines, pass through a point; then the lines are called concurrent lines.

$\left.\begin{array}{l} a_1x+b_1y+c_1=0 \\ a_2x+b_2y+c_2=0 \\ a_3x+b_3y+c_3=0 \end{array}\right\}$ are concurrent then $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

(i.e. point of intersection of two lines has to lie on the third line)

or

there exists, $\lambda_1, \lambda_2, \lambda_3 \in R$ and not all zero such that $\lambda_1L_1+\lambda_2L_2+\lambda_3L_3=0.$

b) Consider the three lines $\left.\begin{array}{l} a_1x+b_1y+c_1=0 \\ a_2x+b_2y+c_2=0 \\ a_3x+b_3y+c_3=0 \end{array}\right\}$

i) These three lines are concurrent if all three lines intersect at a point or any two lines are coincident. (or) all three coincident

ii) These three lines form a triangle if no two lines are parallel or no two lines are coincident or all three lines are not concorrent

12.Distance between a point and the line:

a) Distance from (x_1, y_1) to the line $ax+by+c=0,$ is $\frac{|ax_1+by_1+c_1|}{\sqrt{a^2+b^2}.}$

c) Foot of the perpendicular from (x_1, y_1) to the line $ax+by+c=0,$ is (α, β) given, by

$$\frac{\alpha - x_1}{a} = \frac{\beta - y_1}{b} = \frac{-(ax_1 + by_1 + c)}{a^2 + b^2}.$$

d) image of the point (x_1, y_1) with respect to the line $ax + by + c = 0,$ is (α, β) & it is given by

$$\frac{\alpha - x_1}{a} = \frac{\beta - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}.$$

13.EQUATION OF Parallel lines and distance between them

a). Equation of a line which is parallel to the given line $ax + by + c = 0, \, is \, ax + by + d = 0$

b) distance between the parallel lines $ax + by + c_1 = 0$ & $ax + by + c_2 = 0$ is $\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$

d) Equation of a line which is midway between the parallel lines $ax + by + c = 0$ & $ax + by + d = 0$, is given by

$$ax + by + \frac{c + d}{2} = 0$$

e) Equation of a line which is parallel to the line $ax + by + c = 0$ and "d" distance away from the given line is given by

$$ax + by + c \pm d\sqrt{a^2 + b^2} = 0$$

f). Equation of a line which is perpendicular to the given line $ax + by + c = 0, \, is \, bx - ay + d = 0$

14.Image:

a) Image of the point (x_1, y_1) with respect to the x-axis is $(x_1, - y_1)$

b) Image of the point (x_1, y_1) with respect to the y-axis is $(-x_1, y_1)$.

c) Image of the point (x_1, y_1) with respect to (0,0) is $(-x_1, -y_1)$.

d)Image of the point (x_1, y_1) with respect to the line y=x is (y_1, x_1).

e)Image of (x_1, y_1) with respect to the line $y = x\tan\theta \; is \; (x_1 \cos 2\theta + y_1 \sin 2\theta, \; x_1 \sin 2\theta - y_1 \cos 2\theta)$.

f)Image of (x_1, y_1) with respect to the point y=-x is $(x_1, -y_1)$.

15.Angle between the lines:

b). If is θ the angle, which the line segment, joining the points $(x_1, y_1)(x_2, y_2)$,subtends at the origin then

$$\tan\theta = \frac{\frac{y_1}{x_1} - \frac{y_2}{x_2}}{1 + \frac{y_1 y_2}{x_1 x_2}} = \frac{y_1 x_2 - x_1 y_2}{x_1 x_2 + y_1 y_2}$$

$\cos\theta = \frac{x_1x_2 + y_1y_2}{\sqrt{x_1^2 + x_2^2}\sqrt{y_1^2 + y_2^2}}$. : $\sin\theta = \frac{x_2y_1 - y_2x_1}{\sqrt{x_1^2 + x_2^2}\sqrt{y_1^2 + y_2^2}}$

c). Acute angle between the two lines $a_1x + b_1y + c_1 = 0,\ a_2x + b_2y + c_2 = 0$ is given by $\tan\theta = \left|\frac{a_2b_1 - a_1b_2}{a_1b_2 + b_1a_2}\right|$ = $\left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$

d) Consider the two lines $a_1x + b_1y + c_1 = 0,\ a_2x + b_2y + c_2 = 0$, then

e) lines are parallel $\Rightarrow a_1b_2 = a_2b_1$ and

f) lines are perpendicular $\Rightarrow a_1.a_2 + b_1.b_2 = 0$

g) lines are coincident $\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

i). Equation of the lines, which pass through the point $P(x_1, y_1)$ and making, a given angle α with the given line $y = mx + c$ is $\therefore y - y_1 = \frac{m \pm \tan\alpha}{1 \,?\, m\tan\alpha}(x - x_1)$.

(Where m_1 is slope of the required line)

j). Equation of line, passing through $P(x_1, y_1)$ Which lies on the given line $y = mx + c$, and making equal angles with the given lines "θ " (given) is, obtained by solving (where m_1, m_2 are slopes of the required line)

$$\frac{m_1 - m}{1 + m_1m} = \frac{m - m_2}{1 + mm_2} = \tan\theta.$$

16.Equation of angle bisectors

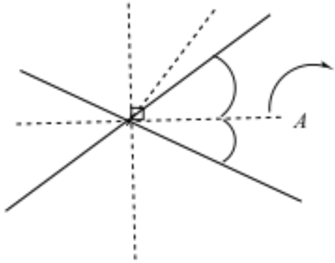
Angle bisector is the collection of all the points, which are at equal distance from the given intersecting lines.

$L_1 \equiv a_1x + b_1y + c_1 = 0,\ L_2 \equiv o_2x + b_2y + c_2 = 0$ are the two lines.

a). Equations of the angle bisectors of the lines are $B_1 = 0\, and\, B_2 = 0$ i.e. $\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$

+ when point (x_1y_1) when 2B's intersect. If $B_1(x_1y_1)\,\&\, B_2(x_2y_2)$ have same sin +ve different -ve

b). Bisector of the angle between the two lines $L_1 = 0,\ L_2 = 0$ which contains a given point A $If\ L_1(A) = positive\ and\ L_2(A) = positive$ or $L_1(A) = negative\ and\ L_2(A) = negative$ is the required



bisector $then\ \frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_2^2}}=+\left(\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right)$ is the required bisector

(i.e. bisector of the region in which the given point A lies)

$If\ L_1(A)=positive\ and\ L_2(A)=negative\ \ then\ \frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_2^2}}=-\left(\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right)$ is the required bisector.

(i.e. bisector of the region in which the given point A lies)

c). Identification of acute / obtuse angle bisector of two given lines $L_1\equiv a_1x+b_1y+c_1=0,\ L_2\equiv o_2x+b_2y+c_2=0$

If $a_1.a_2+b_1.b_2<0$ then $\left(\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right)=+\left(\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right)$ is the acute bisector of the lines

If $a_1.a_2+b_1.b_2<0$ then $\left(\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right)=-\left(\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right)$ is the obtuse bisector of the lines

If $a_1.a_2+b_1.b_2>0$ then $\left(\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right)=+\left(\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right)$ is the obtuse bisector of the lines

If $a_1.a_2+b_1.b_2>0$ then $\left(\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right)=-\left(\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right)$ is the acute bisector of the lines

d). Another way of identifying an acute and obtuse bisectors, is

Let $L_1\equiv a_1x+b_1y+c_1=0,\ L_2\equiv o_2x+b_2y+c_2=0$ and $B_1=0\,and\,B_2=0$ are the bisectors of the given lines.

Take a point P on any one of the given lines $L_1=0, or\ L_2=0$ and find the distance between the point P and the bisector $B_1=0$ (say p) and that of the other bisectors $B_2=0$ is (say q).

If $|p|<|q| \Rightarrow B_1=0$ is the acute angle bisector.

If $|q|<|p| \Rightarrow B_2=0$ is the acute angle bisector.

If $|q|=|p| \Rightarrow L_1=0,\ L_2=0$ are perpendicular lines.

e). Equations of two straight lines $L_1 \equiv a_1x+b_1y+c_1=0,\ L_2 \equiv a_2x+b_2y+c_2=0$

Then, condition for origin to lie in one of the acute angles between the lines is $(a_1a_2+b_1b_2)c_1c_2<0.$

f). Condition for origin to lie in one of the obtuse angles between the lines is $(a_1a_2+b_1b_2)c_1c_2>0.$

18.Equations of median, angular bisector

If $A(x_1,y_1),\ B(x_2,y_2)\ and\ C(x_3,y_3)$ are the vertices of a triangle ABC, D,E,F are the mid points of the sides BC, CA and AB

(I) then equation of median AD, drawn through A is, $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

then equation of median BE, drawn through B is, $\begin{vmatrix} x & y & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

then equation of median CF, drawn through C is, $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

(II) Equation of the angular bisector of $\underline{|A}\ is,\ b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$

19.Family of lines:

a). Family of lines, passing through the point A intersection of two lines $a_2x+b_2y+c_2=0.\ a_1x+b_1y+c_1=0$, is $(a_1x+b_1y+c_1)+\lambda(a_2x+b_2y+c_2)=0$

For different values of λ, we get different lines, passing through the point "A".

b). Equation of a line, passing through the point 'A' intersection $of\ L_1=0\ \&\ L_2=0$ and farthest from $B(x_1,y_1)$, is the line, passing through A, & perpendicular to AB.

Equation of a line, passing through the point 'A' intersection $of\ L_1 = 0\ \&\ L_2 = 0$ and nearest from $B(x_1, y_1)$, is the line, passing through A, & B itself.

c). Parallelogram ABCD with equations of its side AB as $L_1 : a_1x + b_1y + c_1 = 0$, CD as $L_2 : a_1x + b_1y + c_2 = 0$,BC as $L_3 : a_2x + b_2y + d_1 = 0$ and AD as $L_4 : a_2x + b_2y + d_2 = 0$ Then equation of the diagonal BD is given by $L_2L_3 - L_1L_4 = 0$ and equation of the diagonal AC is given by $L_1L_2 - L_3L_4 = 0$

d). A line $ax + by + 1 = 0,$ is such that the algebraic sum of the perpendiculars on it, from a number of points $(x_i, y_i),\ \ (i = 1, 2, ..., n).$ is zero, then the line always passes through a fixed point is $(\bar{x}, \bar{y}) = \left(\bar{x} = \frac{\sum x_i}{n}, \bar{y} = \frac{\sum y_i}{n}\right).$

Some mislaneousresults/examples

a) i)In a triangle ABC, image of the vertex A, with respect to the angular bisector of $\lfloor B$ lies, on the line BC.

ii) Image of the vertex B, with respect to the angular bisector of $\lfloor A$ lies, on the line AC.

iii) Similarly, image of the vertex C, with respect to the angular bisector of $\lfloor B,$ lies, on the line AB.

b) In a triangle ABC, image of the vertex A, with respect to the perpendicular bisectors of the sides AB and AC are Band C respectively

c) Equation of two equal sides of an isosceles triangle ABC, AB, AC are given, then the third side BC, is parallel to the angular bisectors of AB and AC.

d) In a triangle ABC, all the three vertices are given, then, to find out the equation of the internal angle bisector of $\lfloor A,$ Find the point D on BC, which divides BC, in the ratio c:b.(i.e. AB: AC) now write the equation of AD. (or) angular bisectors AB, and AC which contains centroid of the triangle or mid point of BC. (or) use slope form

f) Ceva's theorem:

If the line joining any point "P" to the vertices A,B,C of a triangle meet the opposite sides at D,E and F respectively then $\frac{BD}{DC}.\frac{CE}{EA}.\frac{AF}{FB} = 1$

g) Manelaw's theorem:

If a transversal cuts the side BC,CA and AB of a triangle at D,E and F respectively then $\frac{BD}{DC}.\frac{CE}{EA}.\frac{AF}{FB} = -1$

3). A line cuts x – axis at A, y – axis at B such that AB = l (given), (O-being origin) then the

i) locus of circumcentre of the triangle OAB is $x^2 + y^2 = \frac{l^2}{4}$

ii) locus of orthocentre of the triangle OAB is $x^2 + y^2 = 0 \ (i.e. x = y = 0)$

iii) locus of incentre of the triangle OAB is $x^2 + y^2 = \frac{l^2}{9}$

4) A and B are the fixed points the vertex C of the triangle ABCmoves such that cotA + cotB = c(constant) then locus of the vertex is a line parallel to the line AB.

Example:

5i)

$$\left.\begin{matrix}(a-b)x+(b-c)y+(c-a)=0\\(b-c)x+(c-a)y+(a-b)=0\\(c-a)x+(a-b)y+(b-c)=0\end{matrix}\right\} \begin{matrix}are\ concurrent\ because\\ L_1+L_2+L_3=0\\ here\ \lambda_1=\lambda_2=\lambda_3=1\end{matrix} \quad (or)\begin{vmatrix}a-b & b-c & c-a\\ b-c & c-a & a-b\\ c-a & a-b & b-c\end{vmatrix}=0.$$

5ii) If the lines are $\left.\begin{matrix}ax+by+c=0\\bx+cy+a=0\\cx+ay+b=0\end{matrix}\right\}$ areconcurrent, then

$$\Rightarrow \begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix} = 0 \Rightarrow -(a+b+c)(a^2+b^2+c^2-ab-bc-ca)=0$$
$$\Rightarrow (a+b+c)(a+bw+cw^2)(a+bw^2+cw)=0$$
or

14) In an isosceles triangle, the sum of the distances from any point of the base to the lateral sides is constant

(P is any point on the side BC and h_1, h_2 are the distances between the point and the equal sides AB and AC then $h_1 + h_2 = h$, where h is equal to height of the triangle drawn from B to AC, which is constant)